

A STATISTICAL ANALYSIS OF SOME ESTIMATORS
OF RELIABILITY

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NAVAL POSTGRADUATE SCHOOL

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THESIS

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OF
SOME ESTIMATORS OF RELIABILITY

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by

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ABSTRACT

This thesis presents a computer assisted, comparative analysis of empirical, maximum likelihood and exponential procedures for estimating reliability. The deviations of the estimators from the true reliability, when the underlying failure rate is monotone, are compared using the Kolmogorov-Smirnov family of statistics. The behavior of the distributions of these deviations, for various sample sizes and failure rates is examined. Finally, the considerations of when to use which estimator are discussed.

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I. INTRODUCTION

Assumptions concerning the form of the underlying failure distribution are made in most analyses of reliability problems. When these assumptions are in error the conclusions reached may be grossly in error.

Some alternatives to making assumptions about the form of the underlying failure distribution are maximum likelihood estimation and empirical estimation.

This work presents a comparative analysis of empirical, maximum likelihood, and the often used exponential procedures when the underlying failure distribution has a monotone non-decreasing failure rate. The maximum likelihood estimator is formed under the assumption that the underlying failure rate is monotone non-decreasing. The hypothesis that a sample of failure times comes from a distribution having a monotone failure rate can be tested using the method contained in Ref. 1.

The relative quality of the three estimators was examined using the Kolmogorov-Smirnov family of statistics.

The following notation and definitions are used in the succeeding sections. Let F be a right continuous distribution such that $F(0^-) = 0$.

If F has a density f then $r(t) = \frac{f(t)}{R(t)}$ is the failure rate, where

$R(t) = 1 - F(t)$ is the reliability or the survival probability. Thus

$R(t) = \exp \left[- \int_{-\infty}^t r(z) dz \right]$. A distribution has an increasing failure rate

(IFR) if $r(t)$ is monotone non-decreasing in t , and has a decreasing failure rate (DFR) if $r(t)$ is monotone non-increasing in t .

II. THE ESTIMATORS

A. THE EMPIRICAL ESTIMATOR

The empirical estimator of reliability, $R_{emp}(t)$, based on a sample of n ordered observations $(t_1 \leq t_2 \leq \dots \leq t_n)$ from a failure distribution F is;

$$R_{emp}(t) = 1.0 \quad t < t_1$$

$$R_{emp}(t) = (n-i)/n \quad t_i \leq t < t_{i+1}$$

$$R_{emp}(t) = 0 \quad t \geq t_n$$

B. THE EXPONENTIAL ESTIMATOR

The exponential estimator of reliability based on the above sample is $R_{exp}(t) = \exp[-kt]$, $t \geq 0$, where $1/k$ is the maximum likelihood estimate of the mean of the exponential distribution and is equal to the sample mean.

C. THE MAXIMUM LIKELIHOOD ESTIMATOR

The derivation of the maximum likelihood estimator is dependent upon an assumption that the underlying distribution has IFR or DFR. The IFR case is outlined here. Both the IFR and DFR cases are presented in detail in Ref. 1.

Let $t_1 \leq t_2 \leq \dots \leq t_n$ be a sample of n ordered observations from F , IFR. Using the fact that

$R(t) = \exp \left[-\int_{-\infty}^t r(z) dz \right]$ and $r(t) = \frac{f(t)}{R(t)}$ the log likelihood may be expressed as

$$L = \log(n!) + \sum_{i=1}^n \log f(t_i) = \sum_{i=1}^n \log r(t_i) - \sum_{i=1}^n \int_{-\infty}^{t_i} r(z) dz + \log(n!).$$

The maximization of L , subject to $r(t)$ monotone non-decreasing, yields as an estimator for $r(t)$

$$\hat{r}(t) = \min_{v \geq i+1} \max_{u \leq i} \left[v-u \right] \left[(n-u)(t_{u+1} - t_u + \dots + (n-v+1)(t_v - t_{v-1})) \right]^{-1}$$

$i = 1, 2, \dots, n-1$ and $\hat{r}(t_n) = \infty$. For the remaining values of t , $\hat{r}(t)$ is 0 for $0 \leq t < t_1$, ∞ for $t > t_n$, and constant (right continuous) between observations. The corresponding estimator, $\hat{R}(t)$, is obtained from $\hat{R}(t) = \exp \left[-\int_{-\infty}^t \hat{r}(z) dz \right]$. Reference 1 shows that $\hat{r}(t)$ is a consistent estimator of $r(t)$.

III. PROCEDURES

To evaluate the estimators of $R(t)$, computer simulation (Fortran IV, IBM-360, W. R. Church Computer Center, Naval Postgraduate School) was used to generate samples of failure times, compute the estimators and the statistics used in their evaluation.

Two parent distributions were used, the Weibull and the Erlang. The Weibull distribution has reliability function $R(t) = e^{-bt^a}$ with shape parameter $a > 0$ and scale parameter $b > 0$. The failure rate of the Weibull distribution is $r(t) = abt^{a-1}$. When $a=1$ the distribution is the exponential. For values of $a \geq 1$ the Weibull distribution has IFR.

The Erlang distribution was used to investigate whether the results were dependent upon the parent distribution used. The Erlang distribution has reliability function $R(t) = \int_t^\infty \frac{x^{a-1} b^a e^{-bx}}{(a-1)!} dx$ where $a > 0$, $b > 0$ and a is an integer. The failure rate of the Erlang distribution is $r(t) = \frac{t^{a-1} b^a e^{-bt}}{R(t)}$. When $a=1$ the distribution is the exponential. When $a \geq 1$ the Erlang distribution has IFR and the failure rate is bounded above by b .

Consider a sample of n independent, identically distributed realizations of Weibull failure times. The true reliability of a system with this underlying distribution is $R(t) = e^{-bt^a}$. Let $R_{emp}(t)$ be the empirical estimator of $R(t)$ as computed from this sample and let $D_{emp} = \sup_t |R_{emp}(t) - R(t)|$. Then D_{emp} is a one-sample, two-sided,

Kolmogorov-Smirnov statistic. Computing D_{emp} for each of one hundred independent samples and ordering the results yields an empirical distribution of the statistic D_{emp} .

Repeating this procedure for each estimator gives empirical distributions for the statistics $D_{exp} = \sup_t \left| R_{exp}(t) - R(t) \right|$ and $D_{mle} = \sup_t \left| R_{mle}(t) - R(t) \right|$. Let $F_{demp}(x)$ be the empirical distribution function of the statistic D_{emp} . Similarly, let $F_{dexp}(x)$ and $F_{dmle}(x)$ be the empirical distribution functions of D_{exp} and D_{mle} respectively.

For a given set of Weibull distribution parameters and a given sample size, define;

$$\begin{aligned} D_1^+ &= \sup_x \left[F_{demp}(x) - F_{dmle}(x) \right] \\ D_1^- &= \sup_x \left[F_{dmle}(x) - F_{demp}(x) \right] \\ D_2^+ &= \sup_x \left[F_{dmle}(x) - F_{dexp}(x) \right] \\ D_2^- &= \sup_x \left[F_{dexp}(x) - F_{dmle}(x) \right] \\ D_3^+ &= \sup_x \left[F_{demp}(x) - F_{dexp}(x) \right] \\ D_3^- &= \sup_x \left[F_{dexp}(x) - F_{demp}(x) \right] \end{aligned}$$

These statistics are two-sample, one-sided, Kolmogorov-Smirnov statistics and can be used to test the following types of hypotheses;
 $H_0: F_{demp}(x) = F_{dexp}(x)$ against the alternative $H_1: F_{demp}(x) \geq F_{dexp}(x)$.

To define the critical region for this test, note that $P \left[D_3^+ \leq D^+(\alpha) \right] = 1 - \alpha$, where $D^+(\alpha)$ is the $1 - \alpha$ percentile of the two-sample, one-sided, Kolmogorov-Smirnov statistic distribution.

Reference 2 gives an approximation to the limiting distribution of D^+ as

$$\lim_{\substack{n_1 \rightarrow \infty \\ n_2 \rightarrow \infty}} P \left[D^+ < x \sqrt{\frac{n_1+n_2}{n_1 n_2}} \right] = 1 - \alpha \doteq 1 - e^{-2x^2} \quad \text{where } n_1 \text{ and } n_2 \text{ are}$$

the sample sizes of the two distributions tested.

For a test of the hypothesis $H_0: F_{\text{demp}}(x) = F_{\text{dexp}}(x)$ against the alternative $H_1: F_{\text{demp}}(x) \neq F_{\text{dexp}}(x)$, the two-sample, two-sided Kolmogorov-Smirnov test may be used. The statistic is $D_3 = \max[D_3^+, D_3^-]$. Reference 3 defines the limiting distribution of D and Ref. 2 gives the following approximation

$$\lim_{\substack{n_1 \rightarrow \infty \\ n_2 \rightarrow \infty}} P \left[D < x \sqrt{\frac{n_1+n_2}{n_1 n_2}} \right] = 1 - \alpha \doteq 1 - 2e^{-2x^2}$$

for values of x sufficiently large.

IV. RESULTS

The failure rates examined vary from $r(t) = 1.0$, the case when the underlying distribution is exponential, to $r(t) = 2.139t^2$. In each case the mean time to failure is equal to one.

The important difference in these failure rates is their relative rate of change with t . Throughout the remainder of this discussion the term high failure rate refers to a failure rate with a relatively high rate of change with t . Thus $r(t) = t^2$ is a higher failure rate than $r(t) = t$.

Table I contains the statistics D_1^+ and D_1^- , as defined in Section III. The critical values of these statistics, at a level of significance of .01, are $D_{crit}^+ (.01) = D_{crit}^- (.01) = 0.215$. At a level of significance of .05 $D_{crit}^+ (.05) = D_{crit}^- (.05) = 0.173$. The statistic D can be determined from the table by using the fact that $D = \max [D^+, D^-]$. The critical values of D at levels of significance of .01 and .05 are $D_{crit} (.01) = 0.230$ and $D_{crit} (.05) = 0.192$.

At a level of significance of .01 the hypothesis $H_0: F_{dmle}(x) = F_{demp}(x)$ is rejected in favor of $H_1: F_{demp}(x) \neq F_{dmle}(x)$ when $D_1 > 0.230$. H_0 is rejected in favor of $H_1: F_{demp}(x) \geq F_{dmle}(x)$ when $D_1^+ > 0.215$.

Reversing the sense of the inequality in H_1 and replacing D_1^+ by D_1^- , the test becomes, reject H_0 in favor of $H_1: F_{dmle}(x) \geq F_{demp}(x)$ when $D_1^- > 0.215$.

As all samples of Dexp, Demp and Dmle are of the same size and the Kolmogorov-Smirnov statistics are independent of the distributions being tested, these critical values of D^+ and D^- can be used for all cases presented here.

TABLE I

Observed values of $D_1^+ = \sup_x [F_{demp}(x) - F_{dmle}(x)]$ and $D_1^- = \sup_x [F_{dmle}(x) - F_{demp}(x)]$ for sample size n and failure rate $r(t)$

$r(t)$	$n=10$		$n=15$		$n=20$		$n=25$		$n=30$	
	D_1^+	D_1^-	D_1^+	D_1^-	D_1^+	D_1^-	D_1^+	D_1^-	D_1^+	D_1^-
1.0	0.57	0	0.73	0	0.79	0	0.96	0	0.98	0
$1.287t^{-5}$	0.47	0	0.66	0	0.76	0	0.91	0	0.97	0
$\frac{4t}{2t-1}$	0.43	0	0.70	0	0.85	0	0.92	0	0.97	0
$1.571t$	0.47	0	0.66	0	0.74	0	0.91	0	0.97	0
$1.794t^{1.5}$	0.47	0	0.65	0	0.74	0	0.91	0	0.97	0
$2.139t^2$	0.46	0	0.65	0	0.74	0	0.91	0	0.97	0

In all cases presented in Table I the value of D_1^- is zero and the value of D_1^+ is significantly large which implies that the empirical estimator deviates less from the true reliability than does the maximum likelihood estimator.

As the sample size increases, it is expected that the deviations of both estimators will decrease. The data in Table I indicate that the deviations of $R_{emp}(t)$ are decreasing faster than the deviations of $R_{mle}(t)$.

Before examining some of the sample points of the distributions $F_{demp}(x)$ and $F_{dmle}(x)$, it should be noted that the sample 100th percentile of F_{dmle} , denoted by $Dmle(100)$, is the maximum deviation of $Dmle$ for all samples of fixed size n with the same underlying failure rate. Similarly, $Dmle(1)$ is the minimum deviation and $Dmle(100) - Dmle(1)$ is the sample range of $Dmle$.

As $Demp$ is a one-sample, two-sided Kolmogorov-Smirnov statistic, it is independent of the distributions tested and will vary only with sample size. This is not true of $Dexp$ and $Dmle$.

When the failure rate is $2.139t^2$ and the sample size increases from 10 to 30, $Demp(100)$ decreases from .4120 to .2690, a reduction of about 35% in maximum deviation. Over the same range of sample sizes $Dmle(100)$ decreases from .5426 to .4470, a reduction of about 15% in the maximum deviation. For the same changes in sample size, $Dmle(1)$ varies from .1816 to .1810 while $Demp(1)$ varies from .1210 to .0671. This implies that the entire range of $Demp$ moves toward the origin as the sample size increases, while only the right hand tail of $Dmle$ moves with changes in sample size. When the failure rate is constant the variation in $Dmle(100)$ is from .6956 to .6165 while the variation in $Dmle(1)$ is from .1974 to .1899.

The data in Table II indicate that the deviations of the exponential estimator are stochastically smaller than the deviations of the maximum likelihood estimator for all failure rates tested.

TABLE II

Observed values of $D_2^+ = \sup_x [F_{dmle}(x) - F_{dexp}(x)]$ and
 $D_2^- = \sup_x [F_{dexp}(x) - F_{dmle}(x)]$ for sample size n and failure rate $r(t)$

$r(t)$	$n = 10$		$n = 15$		$n = 20$		$n = 25$		$n = 30$	
	D_2^+	D_2^-	D_2^+	D_2^-	D_2^+	D_2^-	D_2^+	D_2^-	D_2^+	D_2^-
1.0	0	0.88	0	0.94	0	0.93	0	0.99	0	0.99
$1.287t^5$	0	0.84	0	0.82	0	0.88	0	0.95	0	0.98
$\frac{4t}{2t-1}$	0	0.83	0	0.92	0	0.93	0	0.95	0	0.98
$1.571t$	0	0.63	0	0.64	0	0.78	0	0.82	0	0.93
$1.794t^{1.5}$	0.03	0.47	0.08	0.47	0.05	0.57	0.02	0.76	0.01	0.81
$2.139t^2$	0.13	0.30	0.12	0.31	0.09	0.37	0.01	0.51	0.04	0.64

For a given failure rate, the magnitude of D_2^- increases with sample size and for a fixed sample size it decreases as the failure rate increases.

The behavior of the distributions of D_{exp} and D_{mle} are best illustrated graphically. Figures 1 through 4 depict a smoothed form of the empirical distributions as sample size and failure rate vary.

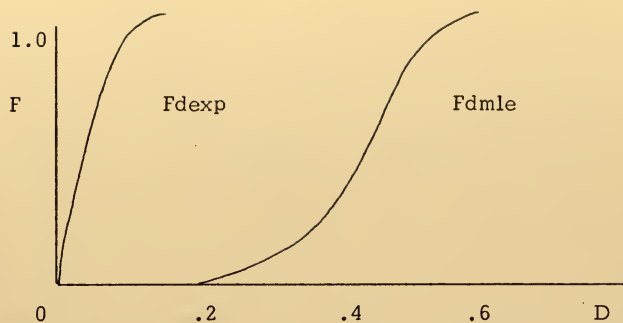


Fig. 1. F_{dexp} and F_{dmle} for $n = 30$ and $r(t) = 1.0$

Note: $D_{exp}(1) = 0.0001$ $D_{exp}(100) = 0.1651$

$D_{mle}(1) = 0.1899$ $D_{mle}(100) = 0.6165$

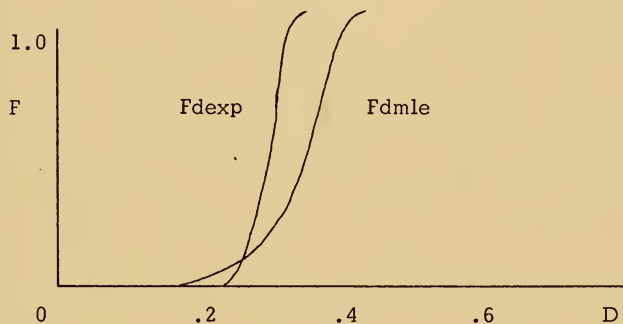


Fig. 2. F_{dexp} and F_{dmle} for $n = 30$ and $r(t) = 2.139t^2$

Note: $D_{exp}(1) = 0.2610$ $D_{exp}(100) = 0.3565$

$D_{mle}(1) = 0.1814$ $D_{mle}(100) = 0.4473$

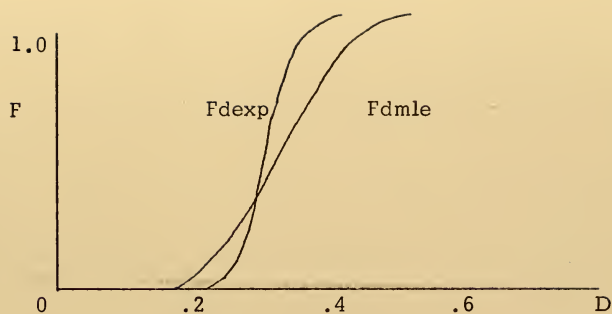


Fig. 3. Fdexp and Fdmle for $n = 10$ and $r(t) = 2.139t^2$

Note: Dexp (1) = 0.2399 Dexp (100) = 0.4291

Dmle (1) = 0.1816 Dmle (100) = 0.5462

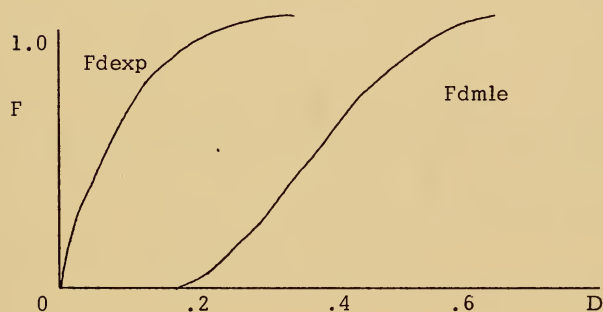


Fig. 4. Fdexp and Fdmle for $n = 10$ and $r(t) = 1.0$

Note: Dexp (1) = 0.0010 Dexp (100) = 0.3571

Dmle (1) = 0.1979 Dmle (100) = 0.6456

As would be expected, the exponential estimator performs best when the failure rate is low and the sample size is large. As the failure rate increases F_{dexp} moves to the right and the range of D_{exp} decreases.

The range of D_{mle} decreases as sample size increases or as failure rate increases. The first percentile of the distribution is relatively insensitive to change in either sample size or failure rate.

For all cases, $H_0: F_{dmle}(x) = F_{dexp}(x)$ can be rejected in favor of $H_1: F_{dexp}(x) \geq F_{dmle}(x)$ at level .01.

The data in Table III indicate that the exponential estimator deviates less from the true reliability than does the empirical estimator when the failure rate is constant or nearly constant. When the failure rate is constant, $D_3^+ = 0$ and D_3^- increases with the sample size, n . This is consistent with the theory as the maximum likelihood estimate of the parameter of the exponential distribution is the reciprocal of the sample mean and $\lim_{n \rightarrow \infty} \bar{x} = \mu$.

TABLE III

Observed values of $D_3^+ = \sup_x [F_{demp}(x) - F_{dexp}(x)]$ and

$D_3^- = \sup_x [F_{dexp}(x) - F_{demp}(x)]$ for sample size n and failure rate $r(t)$

$r(t)$	$n = 10$		$n = 15$		$n = 20$		$n = 25$		$n = 30$	
	D_3^+	D_3^-	D_3^+	D_3^-	D_3^+	D_3^-	D_3^+	D_3^-	D_3^+	D_3^-
1.0	0	0.72	0	0.74	0	0.76	0	0.80	0	0.78
$1.287t^{.5}$	0	0.60	0	0.43	0.05	0.29	0.01	0.23	0.08	0.25
$\frac{4t}{2t-1}$	0	0.63	0.02	0.43	0.02	0.20	0.06	0.26	0.13	0.15
$1.571t$	0.08	0.24	0.30	0.03	0.43	0.06	0.55	0.01	0.66	0
$1.794t^{1.5}$	0.30	0.03	0.63	0	0.66	0.01	0.88	0	0.92	0
$2.139t^2$	0.52	0	0.82	0	0.83	0	0.95	0	0.98	0

As the failure rate becomes larger, D_3^- decreases to zero. D_3^+ increases with both sample size and failure rate. When the failure rate is not constant, D_3^- decreases with sample size.

Figures 5 through 8 show the behavior of F_{demp} and F_{dexp} with changes in failure rate and sample size.

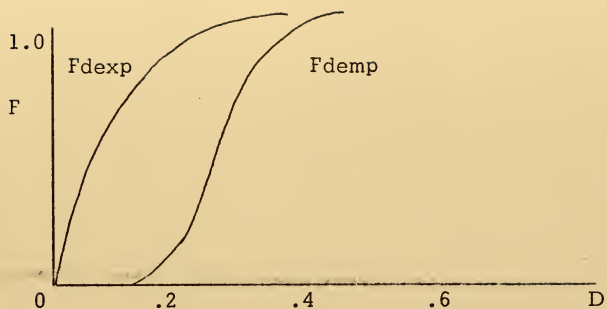


Fig. 5. F_{dexp} and F_{demp} for $n = 1.0$ and $r(t) = 1.0$

Note: $D_{exp}(1) = 0.001$ $D_{exp}(100) = 0.3571$

$D_{emp}(1) = 0.1207$ $D_{emp}(100) = 0.4122$

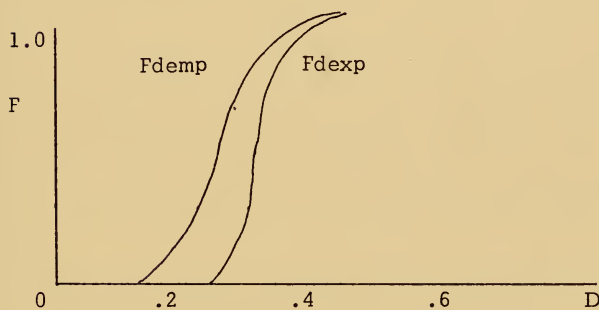


Fig. 6. F_{dexp} and F_{demp} for $n = 10$ and $r(t) = 2.139t^2$

Note: $D_{exp}(1) = 0.2399$ $D_{exp}(100) = 0.4291$

$D_{emp}(1) = 0.1207$ $D_{emp}(100) = 0.4122$

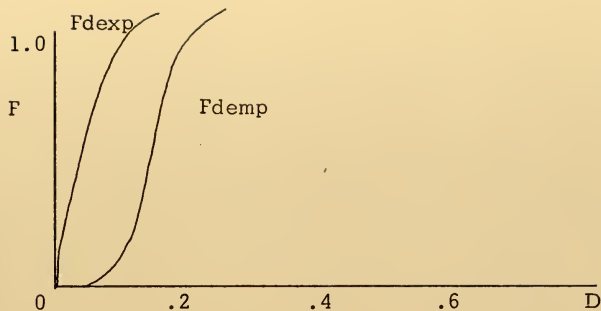


Fig. 7. Fdexp and Fdemp for $n = 30$ and $r(t) = 1.0$

Note: Dexp (1) = 0.0001 Dexp (100) = 0.1651
 Demp (1) = 0.0670 Demp (100) = 0.2690

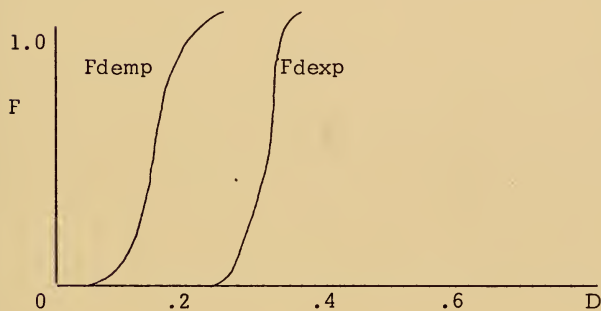


Fig. 8. Fdexp and Fdemp for $n = 30$ and $r(t) = 2.139 t^2$

Note: Dexp (1) = 0.2610 Dexp (100) = 0.3565
 Demp (1) = 0.0670 Demp (100) = 0.2690

Note that the distribution Fdemp is invariant with changes in failure rate and varies only with the sample size. The changes in D_3^+ and D_3^- for a given sample size occur because the distribution Fdexp varies with $r(t)$. As $r(t)$ increases Fdexp moves to the right

for all sample sizes examined. For high failure rates the left end point of F_{dexp} moves away from the origin as sample size increases while the right end point moves toward the origin.

The stochastic ordering of F_{dexp} and F_{demp} varies with the underlying failure rate with $F_{demp} \geq F_{dexp}$ for higher failure rates.

V. CONCLUSIONS

The statistics in Tables I and II support the hypothesis that the empirical and the exponential estimators give better estimates of reliability than does the maximum likelihood estimator for the cases considered.

Comparing the exponential and the empirical estimators, it can be seen that while both sample size and failure rate affect the magnitudes of D^+ and D^- , the failure rate of the underlying distribution is the factor that determines the stochastic ordering of the distributions F_{demp} and F_{dexp} .

The exponential estimator would be the logical choice when the underlying failure rate is constant, or nearly constant, while the empirical estimator performs better when the failure rate is higher.

In considering the question of when to use which estimator it is necessary to investigate the nature of the underlying failure rate.

Reference 4 contains several tests for the validity of the assumption that the underlying life distribution is exponential.

If physical considerations or tests, such as those presented in Ref. 4 substantiate the assumptions that the underlying distribution is exponential, then the exponential estimator of reliability should be used. If the exponential assumption cannot be substantiated, the empirical estimator should give better estimates of the system reliability than the exponential estimator.

Physical considerations of some systems indicate that the failure rate is not monotone but conforms to the bathtub model where the failure rate is initially high but decreasing, relatively constant for the middle age life periods, and increasing with old age. Some electronic components are believed to behave in this manner, experiencing a "burn-in" period at the onset of their working life cycle.

The empirical estimator should be particularly useful in cases where the failure rate is not monotone as no assumptions about the form of the underlying distribution, or its failure rate are required.

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13. ABSTRACT This thesis presents a computer assisted, comparative analysis of empirical, maximum likelihood and exponential procedures for estimating reliability. The deviations of the estimators from the true reliability, when the underlying failure rate is monotone, are compared using the Kolmogorov-Smirnov family of statistics. The behavior of the distributions of these deviations, for various sample sizes and failure rates is examined. Finally, the considerations of when to use which estimator are discussed.			

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KEY WORDS

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Reliability Estimation

Empirical Estimation

Maximum Likelihood Estimation

Statistical Reliability

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A statistical analysis of some estimator



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